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Elastic design of syntactic foamed sandwiches obtained by filling of three-dimensional sandwich-fabric panels

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Abstract

A non-conventional sandwich, made by a fabric panel core filled by a syntactic foam, and by resin-impregnated fiberglass skins, is studied in the elastic range, with the aim of giving guidelines to its minimum weight design. Standard homogenization techniques are employed to compute the elastic moduli of the skins, whereas a specifically developed homogenization method has been used to obtain the elastic moduli of the core. A simple but accurate relationship for computing the shear stiffness of the sandwich was used in conjunction with the well-known formulae for the bending stiffness. Comparisons with both experiments and numerical predictions show good accuracy of both the proposed homogenization methods and the overall stiffness evaluation procedure. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Sandwich panels are often employed in structural applications when weight is a critical issue. The quest for extreme lightness leads to the use, for instance, of honeycomb-core sandwiches, or foamed-core sandwiches. In both cases, a so-called "antiplane" sandwich is obtained in which the purpose of the core is limited to transmitting shear stresses between the skins and to keeping the skin distance approximately constant during the deformation.

These choices, however, may introduce sources of severe structural weakness. A major one is the possibility of debonding between the core and the skins, due, in the case of honeycomb-core sandwiches, to the small contact area between the two layers. A second one relates to strength; in antiplane sandwiches usually the average compressive strength of the core is negligible and, generally, very seldom exceeds the value of 10 MPa, whereas the compressive stress acting on the core itself may sometimes be of one order of magnitude higher. An example occurs in aircraft applications, where a core strength of the order of 30–100 MPa is

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required to carry the load acting on lightweight sandwich panels in proximity of door hinges (Bunn and Mottram, 1993). Also, the elastic stiffness of the core may be an issue.

One way to reduce the risk of delamination between core and skins has been devised in the mid-eighties, when the so-called "sandwich-fabric panels" have been produced in Belgium and in Germany. These are obtained from woven, three-dimensional fabric, impregnated with resin and cured. The fabric is produced by a velvet weaving technique, by skipping the last step of cutting the fabric into two parts. When this fabric is impregnated with resin and cured, a solid panel is obtained, made by two thin skins of resin-impregnated fabric and a core constituted by "piles" of resin-impregnated yarns. A schematic view of this product is given in Fig. 1. A thorough description of the main features of these panels can be found in Van Vuure (1997).

This material is a perfect example of an antiplane sandwich, which does not suffer from problems of delamination as long as the piles connecting the two skins are close enough to each other, and which can function as a proper sandwich as long as the shear stiffness of the core is sufficient to transmit shear stresses and as long as the compressive stiffness of the core is sufficient to prevent relative motion of the skins in the direction orthogonal to their plane. All these conditions are difficult to be met in such a way as to obtain a sandwich usable in structural applications. Moreover, such sandwich has a core with no strength or stiffness in the directions contained in the sandwich plane. Even filling the core of these panels with standard foams does not improve this aspect, as standard foams, since already pointed out, often do not guarantee enough stiffness or enough strength.

From these considerations, the idea arises of filling the core of sandwich-fabric panels with *syntactic* foams, i.e., particulate composite materials made by a matrix of resin and a filler of hollow spheres. Syntactic foams are materials with their own interest, since they exhibit several properties useful for many applications. Among these, the most relevant, within the context of this work, are the low density coupled with a reasonably high stiffness, the relatively high strength, good thermal and water insulation properties, good impact strength and dimensional stability. Syntactic foams are used, for example, as floating material in underwater machines, in aerospace plugs, in the construction of automotive tooling compounds and of ablative heat shields for re-entry vehicles, and even in structural components for submarines and missiles.

The use of syntactic foams as a filler for the core of sandwich-fabric panels allows one to obtain a sandwich, which maintains a low weight without incurring in the previously mentioned drawbacks. In this work, we focus our attention on a sandwich obtained by starting from a fiberglass fabric, impregnated with standard epoxy or polyester resins; the core of the sandwich-fabric panel obtained from this fabric is filled with syntactic foams made again by standard epoxy or polyester resins and by hollow glass microspheres.

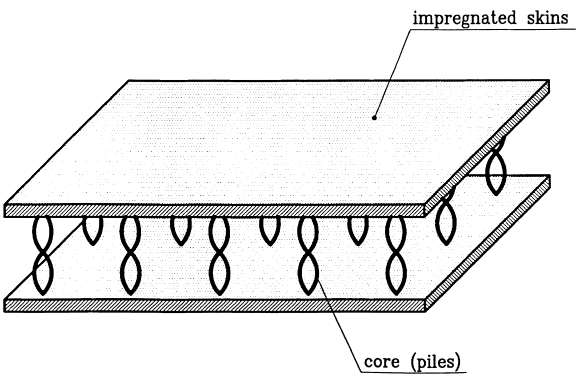

Fig. 1. Sandwich-fabric panel.

Finally, the skins of the sandwich are thickened by applying on their top further layers of resin-impregnated fiberglass.

The purpose of this paper is to illustrate the elastic behavior of this sandwich, in order to understand how its rather involved morphology influences its global stiffness. To do so, a minimum weight sandwich is designed, which has a pre-defined stiffness with respect to a chosen loading condition.

The path followed implies first some elastic homogenization steps, required to reduce this highly non-conventional sandwich to a "standard" three-layered sandwich, in which each layer can be assumed as homogeneous. In particular, there is the need for computing the homogenized elastic moduli of the sandwich skins, made by fiberglass and resin, for computing the homogenized elastic moduli of the syntactic foam, made by resin, glass and air, and, finally, for computing the homogenized elastic moduli of the sandwich core, made by yarns of fiberglass, resin and syntactic foam. The used homogenization methods are briefly summarized in Sections 3 and 4; Section 5 reports a comparison between the analytical estimates and some experimental results.

Having estimates of the elastic moduli of the three layers of the sandwich, it is possible to set up a simple weight optimization procedure following the path indicated, for instance, in Gibson and Ashby (1988). In our case, however, the situation is made difficult by two things: the fact that the sandwich is not of the antiplane type, and the fact that we cannot easily obtain a relationship between the density and the stiffness of the core, as done in Gibson and Ashby (1988). In Section 2, we will describe how we computed the sandwich stiffnesses, and, in Section 6, we will describe the results obtained in minimizing the weight of the sandwich under a given constraint on its overall stiffness.

2. Elastic stiffness of a symmetric sandwich beam

Let us assume to study the elastic stiffness of a sandwich beam of total length l and width B, made of three homogeneous layers, as indicated in Fig. 2. The considered sandwich is symmetric, i.e., its external layers (skins) have identical thickness t. The thickness of the core is indicated with the symbol c. With reference to the terminology used in Allen (1969), we classify our sandwich as one with *thick skins* and a *non-antiplane core*.

The computation of the deflection of a thick skinned non-antiplane sandwich beam can be made using different assumptions, depending on the required degree of accuracy; two possible solutions are suggested in Allen (1969). The first one is approximate and nevertheless very involved, initially formulated for antiplane sandwiches, and extended to the case of non-antiplane core by Allen by adopting the further approximation that the displacement field along the sandwich core is linear. Results obtained using this method, indicated as "Allen's method", will be briefly commented on later.

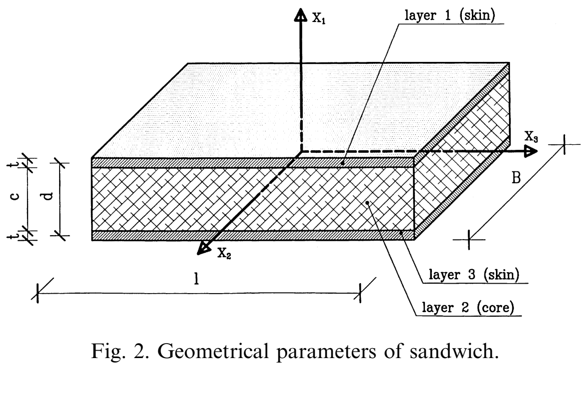

Fig. 2. Geometrical parameters of sandwich.

A second possible solution is based on the Total Potential Energy theorem, which requires the choice of an admissible displacement field along the height of the sandwich section. This approach leads rapidly to very involved computations, and in Allen (1969), it is used only in conjunction with the Ritz method, to obtain approximate solutions, and only for the antiplane core case. We have derived the exact solution, for the three-point bending case, using a kinematics more general than that used by Allen, and in the case of non-antiplane sandwich core (for details see Bardella and Genna, 1999). The results thus obtained will be compared to others in the following, and will be denoted as "Total Potential Energy" results.

Both these methods have some drawbacks, beside their analytical complications, most notably that of failing to yield correct results for special cases, such as that of thin skins, or, worse, that of a homogeneous beam. For this reason, we have chosen to follow a third, extremely simple approach, based on the classic approximate shear force treatment by Jourawsky and on the Navier–Bernoulli homogeneous beam kinematics.

Within this framework, the maximum deflection of a sandwich beam of length l, under general loading and constraint conditions, can be written as follows:

$$\delta = \frac{Pl^3}{Y_1 D} + \frac{Pl}{Y_2 (GA^*)}, \tag{2.1}$$

where P is the total resultant force applied to the beam, D indicates the bending stiffness, G is the shear modulus, A^* is the shear area and Y_1, Y_2 are numerical constants, which depend on both loading and constraint conditions. For instance, in the four-point bending case and for a simply supported beam, which will be used for comparison with experimental results,

$$Y_1 = \frac{32}{\left(1 - \frac{l_p}{l}\right)\left[1 - \frac{1}{3}\left(1 - \frac{l_p}{l}\right)^2\right]}, \tag{2.2}$$

$$Y_2 = \frac{4}{1 - \frac{l_p}{l}}, \tag{2.3}$$

where l_p is the distance between the two concentrated loads, each of magnitude $P/2$. In the particular case of three point bending one has to set $l_p = 0$ in Eqs. (2.2) and (2.3) (i.e., the force P is equal to a concentrated load applied at the beam midspan), which leads to $Y_1 = 48$ and $Y_2 = 4$.

In the case of a thick skinned, non-anti-plane sandwich, the bending stiffness D includes three terms, deriving from contributions of both skins and core, written, with reference to the geometry of Fig. 2, as follows:

$$D = E_{sk}\left(\frac{Bt^3}{6} + \frac{Btd^2}{2}\right) + E_c \frac{Bc^3}{12}, \tag{2.4}$$

where E indicates the Young modulus, subscript sk refers to skin properties, subscript c to core properties and $d = c + t$ indicates the distance between the middle planes of the two skins. This is a completely standard result (Allen, 1969).

A more complicated problem arises from the need for evaluating the sandwich shear stiffness. In fact, the presence of a relatively stiff core and thick skins makes the kinematics of a sandwich beam in bending much different from that of a standard beam, which compels us to take into account the shear deformability in a rather complicated way. Such phenomenon is the cause of the involved methods mentioned above; however, if one simply assumes that a plane sandwich section remains plane during the deformation, by developing the classical beam analysis for shear stresses due to shear force, one can compute the shear stresses in the three layers of the sandwich. Thereafter, one can obtain the "equivalent" shear stiffness of the sandwich by equating the expressions of the work of deformation in the sandwich beam and in the homogeneous equivalent beam. After some lengthy algebra, the following results are thus obtained:

$$(GA^*) = \frac{\frac{5}{6}}{\frac{4}{A_{sk}G_{sk}(1+\alpha_s)} + \frac{1}{A_c G_c(1+\alpha_c)}}, \tag{2.5}$$

where $A_{sk} = 2tB$ is the area occupied by the skins in a beam section, $A_c = cB$ is the core area and the interaction coefficients α_s and α_c are defined as follows:

$$\alpha_s = \frac{\frac{c^6}{n^2} + \frac{12c^5 t}{n} + c^4t^2(36 + \frac{24}{n}) + c^3t^3(144 + \frac{16}{n}) + 230c^2t^4 + 167ct^5 + 48t^6}{t^4(10c^2 + 25ct + 16t^2)}, \tag{2.6}$$

$$\alpha_c = \frac{\frac{c^5 t}{3n} + c^4t^2(1 + \frac{7}{3n}) + 2c^3t^3(7 + \frac{4}{3n}) + 35c^2t^4 + 32ct^5 + \frac{32t^6}{3}}{c^2\left(\frac{c^4}{6n^2} + \frac{5c^2t(t+c)}{3n} + 5t^2(c+t)^2\right)}, \tag{2.7}$$

where n is the ratio between the Young moduli of skins and core:

$$n = \frac{E_{sk}}{E_c}. \tag{2.8}$$

Despite the conceptual simplicity of this computation, we have not been able to find any reference to it in the literature. In order to verify the accuracy of Allen's method, of the Total Potential Energy method and of results (2.5)–(2.8), we have run some finite element simulations, on an arbitrary geometry of sandwich beam, for several values of ratios n and several values of ratios t/c. The relevant results are shown in Fig. 3, where percentage errors given by the three approaches (Allen, Total Potential Energy, Jourawsky), computed using the finite element solution as reference solution, are plotted as function of the ratio t/c for some values of coefficient n. It is apparent that Allen's method gives the worst results in all cases (it fails badly, in particular, for ratios $t/c \to 0$) and that even the Total Potential Energy method, in which the "exact" sandwich beam kinematics has been inserted, is not really accurate, owing to the inconsistency between the use of a trilinear kinematic model (Allen, 1969) and the strain field implied by Jourawsky's approach. The results given by Eqs. (2.5)–(2.8) seem to be an acceptable approximation of finite element results over the whole range of variables considered.

In the following, therefore, we will use Eqs. (2.4) and (2.5)–(2.8) to compute the equivalent elastic stiffnesses of a sandwich beam. In particular, we will need them in Sections 5 and 6. In Section 6, we will prescribe Eq. (2.1) as a constraint during the weight optimization of the sandwich. During this process, we will also use, as a simpler approximation, both the expressions of the bending and shear stiffness, both for thin skins and antiplane core:

$$D = E_{sk}\frac{Btd^2}{2}, \tag{2.9}$$

$$(GA^*) = G_c Bd. \tag{2.10}$$

3. Homogenization of the syntactic foam

The application of the equations providing the stiffness of a sandwich beam, briefly summarized in the previous section, requires the knowledge of the values of the elastic moduli of the layers of the sandwich,

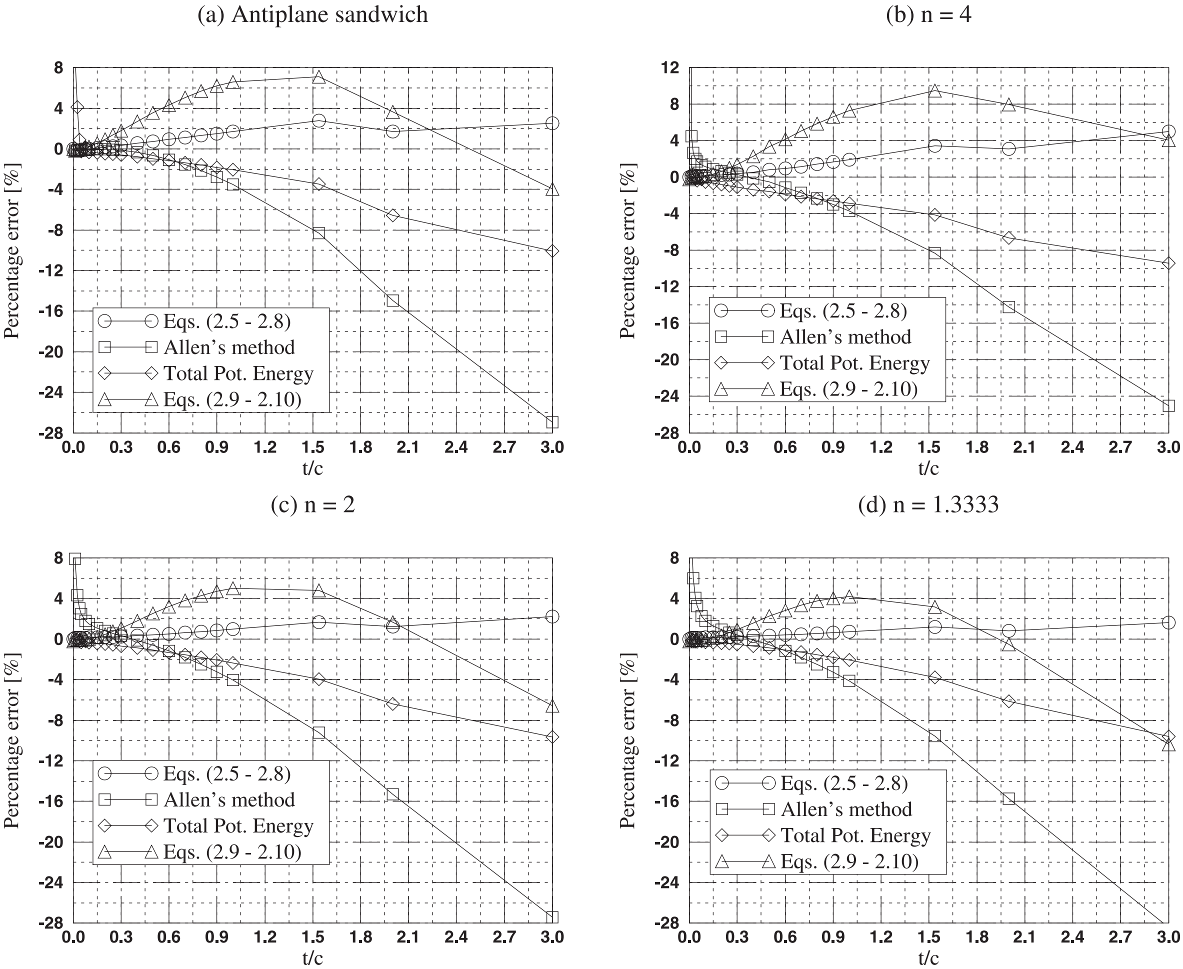

Fig. 3. Percentage errors of methods for the computation of sandwich beam deflections.

i.e., of the skins and of the core, seen as homogeneous materials. As, in reality, each layer of the studied sandwich is made up of a composite material, we need to estimate the "equivalent" elastic moduli of these layers by means of suitable homogenization techniques.

For this sandwich, there is the need for homogenizing at two different geometric scales. The first one refers to the syntactic foam, which fills the space left, in the core of the sandwich, by the resin-impregnated yarns of the sandwich-fabric panel. The syntactic foam is itself a composite material, whose inclusion size is about 5–100 μm. The second scale is that defined by the inclusions of both skins and core; in the skins, the inclusions are the glass fibers, whereas in the core, the inclusions are defined by the glass yarns surrounded by resin. Here, the geometric scale of the inclusion is about the diameter of one glass yarn, i.e., according to Van Vuure (1997), of the order of 0.1–0.5 mm.

As the geometric scales of the syntactic foam and of the sandwich core are different by roughly one order of magnitude, it is possible to compute the homogenized elastic moduli of the core by means of a two-step homogenization technique: the first one, to compute the equivalent elastic moduli of the syntactic foam, and the second one to compute those of the core itself. The second step will be described in Section 4.

The first step is somewhat involved, owing to the presence of the hollow microspheres as a filler of the foam. This fact makes it difficult to particularize classical homogenization methods, which, in this situation, tend to perform poorly. We have been able to find only four references specifically concerned with the homogenization of syntactic foams (Lee and Westmann, 1970; Huang and Gibson, 1993; Nielsen and Landel, 1994; Hervé and Pellegrini, 1995). The methods proposed in the first three references do not yield accurate results over a wide range of material parameters.

The paper by Hervé and Pellegrini (1995) gives indeed a complete theoretical treatment of the problem of determining the elastic constants of a material containing spherical coated holes, considering the extremely general case of n spherical layers. In Hervé and Pellegrini (1995), however, the authors particularize their full analytical solution only to the case of standard foams.

There are two more difficulties to be taken into account when trying to compute the homogenized elastic moduli of real syntactic foams. The first one is the possible presence of "unwanted" air, due to the production modalities, which cannot be taken into account by the formulae of Hervé and Pellegrini. The second is the scatter of the filler geometry – in particular, the gradation of the ratios a/b between the inner and outer diameter of the spheres, which may have a strong effect on the final stiffness of the syntactic foam. Also, this feature cannot be taken into account by using the results of Hervé and Pellegrini.

We have therefore extended the model of Hervé and Pellegrini, limiting ourselves to the case of syntactic foams, i.e., by setting $n = 3$ in their n-layered inclusion model, by considering a representative volume element (RVE) made by N different composite sphere types (Hashin, 1962), each characterized by a specific value of the ratio a/b. In this way, we can consider both the actual gradation of the filler and the presence of a void phase entrapped in the matrix, corresponding to setting $a/b = 1$. All the results obtained by this model are reported in Bardella and Genna (2000).

It might be worth commenting on the possibility of using a simpler sequential homogenization technique to take into account the presence of voids in the matrix. Such a procedure – first homogenize the voids with the matrix material, then use, for instance, Hervé and Pellegrini result to homogenize the inclusions with the weakened matrix – can reasonably be used when the geometric scales of voids and inclusions differ by at least one order of magnitude. In our syntactic foams, unfortunately, this is not the case, since the used microspheres, as said, have an average diameter of 5–100 μm, and the void bubbles present in the matrix have a comparable size (Bardella and Genna, 2000). Therefore, it is inconsistent to first homogenize the voids and then the hollow inclusions, and the only proper way to proceed is to homogenize the three phases simultaneously.

In the present paper, we have not tackled the study of the effect of the filler gradation on the global stiffness of the sandwich, expected to be non-significant. Also, in the design of the "optimal" sandwich, we have assumed, for the sake of simplicity, that no "unwanted" voids are present into the syntactic foam. In this case, the formulae we have used to compute the homogenized elastic moduli of the syntactic foam are those given by Hervé and Pellegrini (1995) for $n = 3$; since their results are not particularized to the case $n = 3$ and are therefore rather involved, here we report the explicit formulae we have used in the process of identifying the lightest sandwich. When comparing experimental with analytical results, however, we had to resort to the complete results reported in Bardella and Genna (2000), as, in this case, the presence of voids in the matrix had a significant effect and could not be neglected.

In the homogenization technique, the syntactic foam is considered as a macroscopically homogeneous and isotropic material; this assumption is justified by the statistically random distribution of both position and geometry of the inclusions, clearly visible in the image shown in Fig. 4, obtained by the scanning electron microscope. The assumption of perfect bond between matrix and filler has always been made.

The used physical model is similar to that proposed by Christensen and Lo (1979) to homogenize particulate composites with solid inclusions. As done by Hervé and Pellegrini (1995), we extended the results of Christensen and Lo by computing elastic solutions of a four-phase model, illustrated in Fig. 5, which consists of a hollow "composite sphere", surrounded by an infinite medium of arbitrary material.

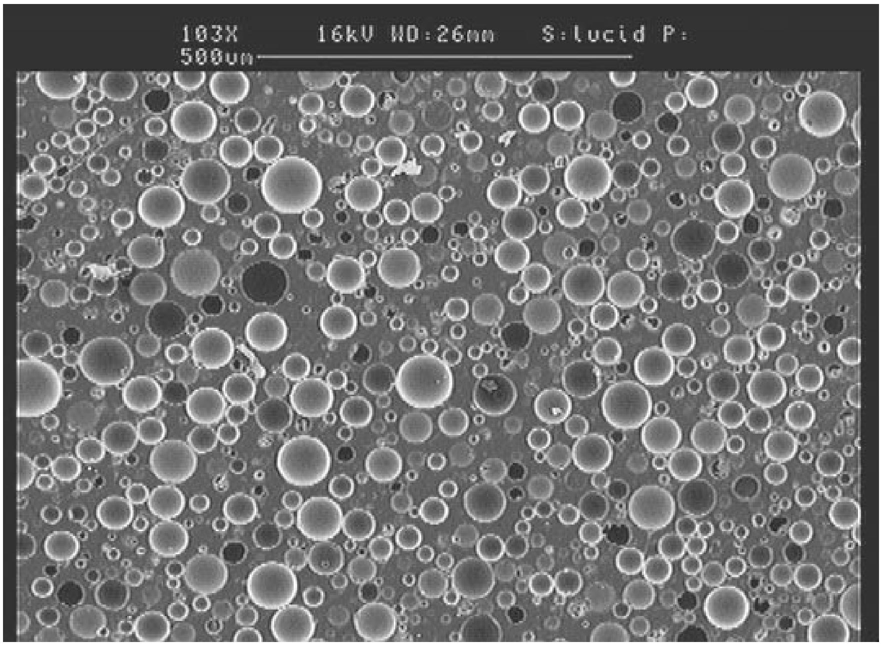

Fig. 4. Scanning electron microscope image of the syntactic foam microstructure.

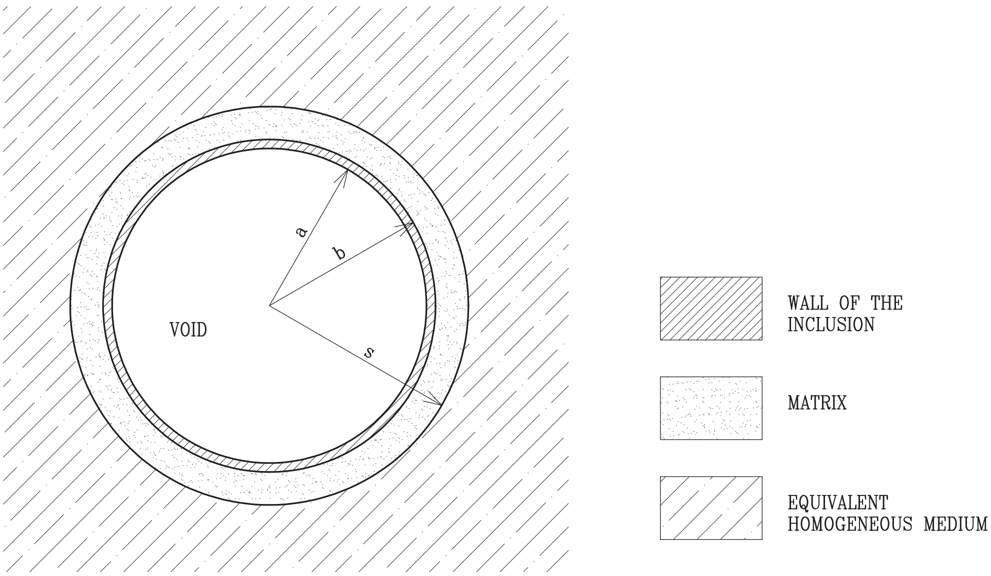

Fig. 5. The four-phase model used for the homogenization of the syntactic foams.

Going through the relevant average operations, over the various phases of the composite and after some lengthy algebra, one arrives at the following form for the "best" estimate G_0 of the shear modulus of a syntactic foam:

$$\left(40H_1\frac{a^5}{s^5}\right)\left(\frac{G_0}{G_m}\right)^2+\left(2F_4-F_2-8(F_1+3F_3)\frac{a^5}{s^5}\right)\left(\frac{G_0}{G_m}\right)+\frac{F_2F_3-F_1F_4}{H_1}=0, \tag{3.1}$$

where G_m is the shear modulus of the matrix (i.e., the shear modulus of the epoxy resin), and whose coefficients F_1, F_2, F_3, F_4 and H_1 are given in Appendix A.

The significant root of the quadratic Eq. (3.1) for G_0 is positive (i.e., greater than the value of the shear modulus of the void), and lower than the highest value between the shear modulus of the matrix and that of the inclusion.

The solution for the homogenized bulk modulus is (Lee and Westmann, 1970)

$$K_0=K_m\frac{\delta\left(1+\frac{b^3}{s^3}\gamma\right)+\kappa\left(1-\frac{b^3}{s^3}\right)\gamma}{\delta\left(1-\frac{b^3}{s^3}\right)+\kappa\left(\gamma+\frac{b^3}{s^3}\right)}, \tag{3.2}$$

where

$$\gamma=\frac{4G_m}{3K_m},\qquad \delta=\frac{4G_i}{3K_m}\left(1-\frac{a^3}{b^3}\right),\qquad \kappa=\frac{4G_i}{3K_i}+\frac{a^3}{b^3} \tag{3.3}$$

in which index i refers to properties of the inclusion's wall (glass, in our case).

The accuracy of the predictions given by this homogenization technique is shown in Fig. 6, which refers to a syntactic foam made by an epoxy resin of the type DGEBA, with hardener DDM and microspheres

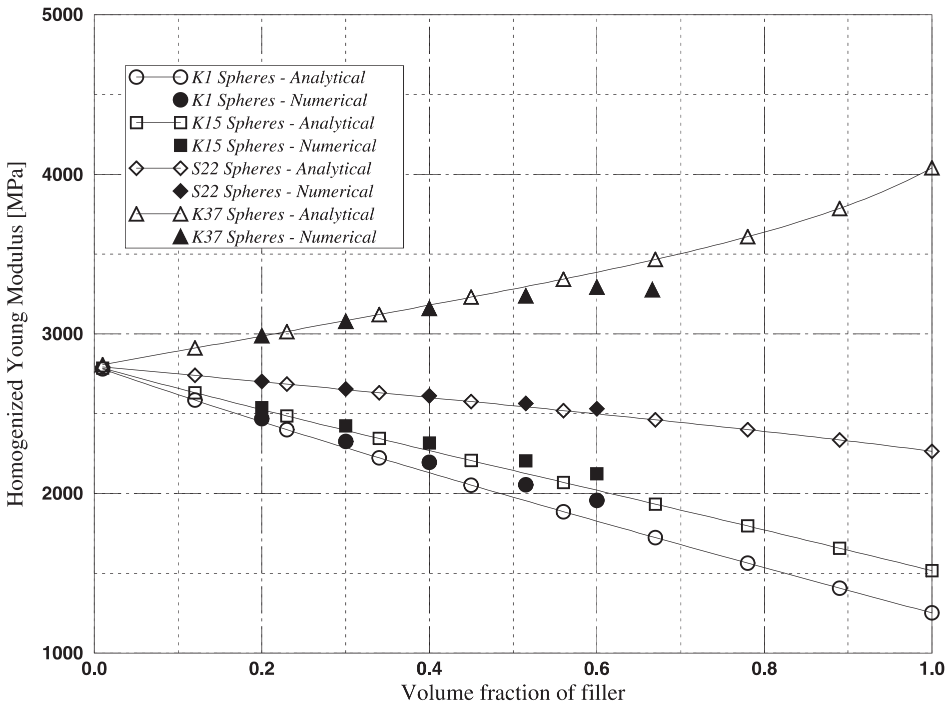

Fig. 6. Comparison of analytical estimates of the homogenized Young modulus of the syntactic foam with numerical simulation results.

taken from the industrial batch produced by 3M and described in the report 3M Italia (1993). All the data about these materials are furnished in Bardella and Genna (2000). Fig. 6 compares the analytical predictions with numerical ones, obtained by the finite element analysis of a unit cell of material (Bardella and Genna, 2000), for several different types of microspheres; for all cases, the analytical results agree reasonably well with the numerical ones and, more important, for all cases, the analytical results are able to correctly predict the slope of the homogenized moduli curve as a function of the volume fraction of filler.

4. Homogenization of the fiber-reinforced resin and of the sandwich core

The computation of the equivalent elastic moduli of the sandwich layers, assuming to know the values of the elastic moduli of glass fibers, resin and syntactic foam all seen as homogeneous isotropic materials, at the length scale of the sandwich, can be performed exploiting results available in the literature. However, since the morphology of the studied sandwich is extremely complex, the use of standard results requires some interpretation.

With reference to the cartesian reference system x_1, x_2, x_3, shown in Fig. 2, both the skins and the core can be approximately considered as transversely isotropic, where every plane containing axis x_1 (or any other axis parallel to it) is a plane of material symmetry.

This statement is sufficiently accurate in the case of the core, where the geometry of the yarns connecting the two skins does not give rise to preferential directions in the plane x_2–x_3 (the yarns may be placed at different distances in the warp direction and in the weft direction; a moderate in-plane anisotropy can be expected from this arrangement, but it can definitely be neglected when considering the overall behavior of the sandwich).

Actually the resin-impregnated glass yarns – the piles in the core – can be shaped in various ways, from straight, if the sandwich-fabric panel is subjected to a process called "adhesive foil stretching" (Van Vuure, 1997), to S shapes and C shapes, if the panel is not stretched. Also, the piles, during the impregnation process, group together, to form the so-called "pillars"; these, depending on the production process, can be inclined with respect to axis x_1, and are often designed on purpose with an inclination of $\pm 45°$. All these possibilities complicate somewhat the geometrical description of the sandwich core; however, for most practical purposes, the assumption of considering straight pillars only can be considered acceptable.

Of the possible methods, for the computation of the homogenized elastic moduli of transversely isotropic composites, we have used that proposed by Walpole (1969), which actually considers the single phases of the composite as transversely isotropic as well.

The use of Walpole's method for our materials, however, requires some interpretation, as Walpole gives bounds for all the elastic moduli of the composite, and we have to understand what bounds are to be chosen in relation to the morphology of both the skins and the core of the studied sandwich.

Our composite materials – both the skins and the core of the sandwich – are transversely isotropic only because of the geometrical layout of their constituents, which can be considered, individually, as isotropic. In this case, the bounds given by Walpole can be applied for any possible geometrical arrangement of the single phases which, being individually isotropic, can always be imagined as having an axis of transverse isotropy coinciding with the global axis of transverse isotropy, x_1.

For the purpose of this work, we need to estimate the values of the in-plane Young and shear moduli of both the skins and the core, hereafter indicated by the symbols E_p and G_1, respectively.

In the core, where the stiff glass fibers are aligned with the direction of the axis of transverse isotropy, when considering the in-plane behavior (x_2–x_3), the stiff inclusion phase is not connected and evidently offers little contribution to the global stiffness. For this reason, we have chosen to use the lower bound given by Walpole for the Young modulus. The reasoning becomes more difficult in the case of the shear modulus; therefore, in order to get a better understanding, we also computed the analytical estimates for a transversely isotropic material given in Hervé and Zaoui (1995), which apply to a material, whose morphology is exactly that of the core of our sandwich (but not that of the skins). For both the Young's and the shear modulus, we obtained results practically coinciding with Walpole's lower bounds. Therefore, in order to avoid the description of a further set of formulae in this paper, we decided to use Walpole's lower bounds to compute the elastic moduli of the core.

In the case of the skins, the presence of a layer of fabric, lying in the middle planes of skin, parallel to plane x_2–x_3, introduces a moderate amount of anisotropy in the plane. Such anisotropy is shown by laboratory tests performed on the skins only (Bardella and Genna, 1999), which indicate values of Young's moduli, for a particular choice of basic ingredients, in the ratio $E_2/E_3 \approx 1.20$. This ratio is closer to unity than one might expect, considering the strong orthotropy of the fiberglass, fabric from which the sandwich-fabric panel is obtained; one needs to recall that, in the final sandwich, the skins are reinforced by the application, during the curing phase, of further layers of resin-impregnated fiberglass, in which the glass fibers are randomly oriented. These layers, whose thickness is significantly greater than that of the skin obtained by impregnating the original fiberglass fabric, are in themselves almost exactly transversely isotropic around axis x_1. The final result is that, as said, of a moderate anisotropy of the skins in their plane, which contains the stress components of interest to us. However, for all practical purposes, in the sequel of this work, we will assume that the skins are also transversely isotropic around axis x_1.

For the skins, unlike the case of the core, the internal morphology sees the stiff glassy phase aligned in the skin middle plane, i.e., close to a "parallel" arrangement with the matrix phase. This suggests the use of Walpole's upper bound for Young's modulus of the skins.

The interpretation of the values of the shear modulus is again more difficult. As we could not find any other reference to specific methods to compute such constant, we decided to use, as an estimate of the value of the skin shear modulus, the average value between the lower and the upper bound. On the other hand, the value of the shear modulus of the skins has almost no relevance on the overall stiffness of the sandwich, and we do not expect this assumption to be the cause of significant errors.

A macroscopically transversely isotropic material is characterized, in the linear elastic range, by five independent elastic constants, which can be chosen in different ways. Walpole (1969) gives bounds for all the following constants:

- $\kappa_{23}^{(0)}$, plane strain bulk modulus, with reference to the isotropy plane x_2–x_3, defined by the following strain field: $\varphi_{11} = 0$, $\varphi_{22} = \varphi_{33} = \varphi$ and by the relationship $\sigma_{22} = \sigma_{33} = \sigma = 2\kappa_{23}^{(0)}\varphi$;
- $G_{23}^{(0)}$, shear modulus in the isotropy plane, defined by the following relationship: $\sigma_{23} = 2G_{23}^{(0)}\varphi_{23}$;
- $G_1^{(0)}$, shear modulus, defined by any of the two following relationships: $\sigma_{12} = 2G_1^{(0)}\varphi_{12}$, $\sigma_{13} = 2G_1^{(0)}\varphi_{13}$;
- $L^{(0)}$, cross modulus, as defined by Hill (1964);
- $N^{(0)}$, longitudinal modulus in the direction of axis x_1 in the absence of transverse deformation,

where the superscript (0) indicates homogenized values for the equivalent homogeneous material.

The meaning of the last two constants is defined by writing a transversely isotropic constitutive law in the following way:

$$\tfrac{1}{2}(\sigma_{22} + \sigma_{33}) = \kappa_{23}^{(0)}(\varphi_{22} + \varphi_{33}) + L^{(0)}\varphi_{11}, \tag{4.1}$$

$$\sigma_{11} = L^{(0)}(\varphi_{22} + \varphi_{33}) + N^{(0)}\varphi_{11}, \tag{4.2}$$

$$\sigma_{22} - \sigma_{33} = 2G_{23}^{(0)}(\varphi_{22} - \varphi_{33}), \qquad \sigma_{23} = 2G_{23}^{(0)}\varphi_{23}, \tag{4.3}$$

$$\sigma_{12} = 2G_1^{(0)}\varphi_{12}, \qquad \sigma_{13} = 2G_1^{(0)}\varphi_{13}. \tag{4.4}$$

Here we write explicitly the specialization of Walpole's results to the case in which the single components of the composite are isotropic. In this case, each phase is characterized by two elastic constants only, instead of the five used by Walpole. In order to re-write Walpole's equations using his constants, we need to make the following replacement of elastic constants of each single phase ζ:

$$L^{(\zeta)} = \kappa_{23}^{(\zeta)} - G_{23}^{(\zeta)}, \tag{4.5}$$

$$N^{(\zeta)} = \kappa_{23}^{(\zeta)} + G_{23}^{(\zeta)}, \tag{4.6}$$

$$G_1^{(\zeta)} = G_{23}^{(\zeta)}. \tag{4.7}$$

In this way a single isotropic phase ζ is characterized only by two elastic constants $G_{23}^{(\zeta)} = G_\zeta$, and $\kappa_{23}^{(\zeta)} = \kappa_\zeta = \lambda_\zeta + G_\zeta$, where λ_ζ is the second Lamé constant.

The formulae to compute lower (superscript low) and upper (superscript up) bounds to the five elastic moduli for a transversely isotropic composite made by m isotropic phases, each of volume fraction c_r, with $\sum_{r=1}^{m} c_r = 1$, are the following, where to obtain lower bounds one needs (i) to replace the symbol ★ with the symbol low in the homogenized moduli and (ii) to give to the constants indicated with subscript ★, at the right-hand sides, the lowest values of the same constants among the single phases; to obtain upper bounds, one needs to replace the symbol ★ with the symbol up in the same way:

$$\kappa_{23}^{\star} = \left(\sum_{r=1}^{m} \frac{c_r}{\kappa_r + G_\star} \right)^{-1} - G_\star, \tag{4.8}$$

$$G_{23}^{\star} = \left(\sum_{r=1}^{m} \frac{c_r}{G_{\mathrm{r}} + \dfrac{G_\star \kappa_\star}{\kappa_\star + 2G_\star}} \right)^{-1} - \frac{G_\star \kappa_\star}{\kappa_\star + 2G_\star}, \tag{4.9}$$

$$G_1^{\star} = \left(\sum_{r=1}^{m} \frac{c_r}{G_{\mathrm{r}} + G_\star} \right)^{-1} - G_\star, \tag{4.10}$$

$$L^{\star} = \frac{\displaystyle\sum_{r=1}^{m} \frac{c_r(\kappa_r - G_{\mathrm{r}})}{\kappa_r + G_\star}}{\displaystyle\sum_{r=1}^{m} \frac{c_r}{\kappa_r + G_\star}}, \tag{4.11}$$

$$N^{\star} = \sum_{i=1}^{m} c_i(\kappa_i + G_{\mathrm{i}}) + \frac{\displaystyle\sum_{r=1}^{m} \left(\frac{c_r(\kappa_r - G_{\mathrm{r}})}{\kappa_r + G_\star} \sum_{s=1}^{m} \frac{c_s(\kappa_s - G_{\mathrm{s}} - \kappa_r + G_{\mathrm{r}})}{\kappa_s + G_\star} \right)}{\displaystyle\sum_{r=1}^{m} \frac{c_r}{\kappa_r + G_\star}}. \tag{4.12}$$

Of course, for the case of the core homogenization, one must set $m = 3$ (fiberglass, resin and syntactic foam) and, for the homogenization of the skins, $m = 2$ (fiberglass and resin).

The expression of the in-plane Young modulus $E_{\mathrm{p}}^{(0)}$, previously defined, is the following:

$$E_{\mathrm{p}}^{(0)} = \frac{4G_{23}^{(0)}\kappa_{23}^{(0)}}{\kappa_{23}^{(0)} + G_{23}^{(0)} + \dfrac{(L^{(0)})^2 G_{23}^{(0)}}{\kappa_{23}^{(0)} N^{(0)} - (L^{(0)})^2}}. \tag{4.13}$$

It is not easy to understand how to combine the upper and lower bounds (4.8)–(4.12) to obtain upper and lower bounds of the in-plane Young modulus (4.13), since this dependence varies with the number of the phases and their relative stiffnesses. For the sandwich studied here, and for the common values of the elastic moduli of the various phases, we have found that the following relationships always give lower and upper bounds of the in-plane Young modulus E_p:

$$E_p^{low} = \frac{4G_{23}^{low}\kappa_{23}^{low}}{\kappa_{23}^{low} + G_{23}^{low} + \dfrac{(L^{up})^2 G_{23}^{low}}{\kappa_{23}^{low}N^{low} - (L^{up})^2}}, \quad (4.14)$$

$$E_p^{up} = \frac{4G_{23}^{up}\kappa_{23}^{up}}{\kappa_{23}^{up} + G_{23}^{up} + \dfrac{(L^{low})^2 G_{23}^{up}}{\kappa_{23}^{up}N^{up} - (L^{low})^2}}. \quad (4.15)$$

Owing to the microstructural arrangement of the core and of the skins of the sandwich under investigation, for the reasons illustrated at the beginning of this section, we will use the lower bound (4.14) as an estimate of Young's modulus of the core, and the upper bound (4.15) as an estimate of Young's modulus of the skins.

An indication about the accuracy of this choice has been sought both for the skins alone and for the core of the sandwich. For the skins, experimental results have been obtained by means of uniaxial tension tests performed in our Laboratory (Bardella and Genna, 1999). The skins were made of an epoxy resin with Young's modulus $E_r = 3700$ MPa and Poisson's coefficient $\nu_r = 0.4$, with a volume fraction $f_g = 0.342$ of glass fibers, with Young's modulus $E_g = 73\,000$ MPa and Poisson's coefficient $\nu_g = 0.23$. The average values of the in-plane Young moduli for this material are $E_p^a = 13\,720$ MPa in the warp direction and $E_p^e = 16\,360$ MPa in the weft direction; with the same data, the upper bound estimate (4.15) is $E_p = 16\,300$ MPa, in reasonable agreement with the experimental results.

We could not perform any experimental test on the core material of the sandwich, made of the syntactic foam mixed with the resin-impregnated glass yarns. Therefore, we have tested the accuracy of the predictions given by the lower bound (4.14) by means of numerical simulations, performed on a three-dimensional unit cell of the material. The glass yarn, surrounded by resin, has been considered as a cylindrical element of circular cross section; its inner part is made of glass, and a circular ring of resin has been added around it; the volume fraction of glass, in this cylindrical element, is equal to that of the glass in the skins, i.e., $f_g = 0.342$.

The unit cell is prismatic, of height equal to c (thickness of the core) with a square base, whose sides are chosen to be equal to the average distance between the piles in the sandwich-fabric panel. Periodicity boundary conditions have been prescribed on the vertical sides of the prism. The materials here are the same resin and glass used for the skins, and microspheres taken from the 3M industrial batch of the type K1 (3M Italia, 1993).

Fig. 7 shows the used mesh, and Fig. 8 compares both the analytical predictions (4.14) and (4.15) with the numerical results, for volume fractions of impregnated glass fibers ("pillars") in the core ranging from $f_p = 0$ to $f_p = 0.2$; actual values of this parameter, for the studied sandwiches, are of the order of $f_p \approx 0.05$. As expected, there is a reasonable agreement of the numerical results with the lower bound estimates.

5. Verification of the model by comparison with experimental results

The homogenization methods described in the previous sections give the values of the elastic moduli of both the skins and the core of the sandwich, considered as homogeneous materials, as function of the geometric and material parameters of their constituents. Therefore, we can treat the sandwich, from now

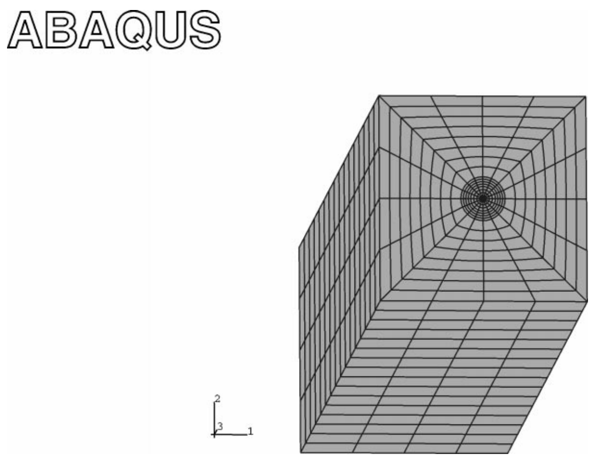

Fig. 7. The unit cell mesh used for the numerical evaluation of the elastic moduli of the sandwich core.

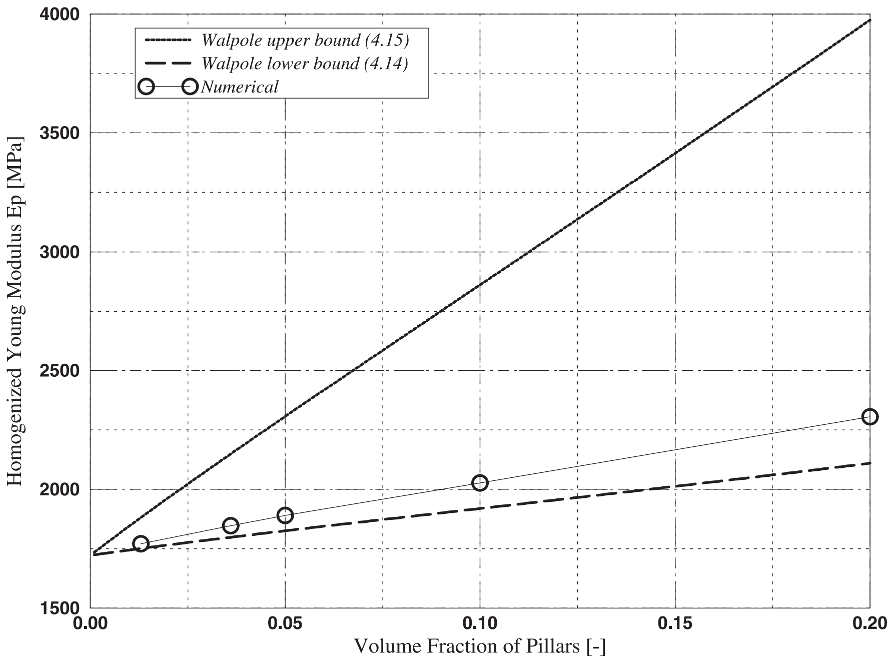

Fig. 8. Comparison of Walpole's bounds on in-plane Young's modulus of the sandwich core (Eqs. (4.14) and (4.15)) with predictions of numerical simulations.

on, as a standard sandwich, and use the relationships summarized in Section 2 to compute its stiffness in bending.

In this section, we compare the results of the analytical predictions with some experimental results obtained at the Politecnico of Milano (Maier, 1998). These results refer to a single type of sandwich, and to two different testing modalities: three-point bending and four-point bending.

The basic materials of the sandwich – resin and glass – are of the type already described in Section 4:

- epoxy resin, Young's modulus $E_r = 3700$ MPa, Poisson's coefficient $\nu_r = 0.4$, density $\rho_r = 1.15$ g/cm^3;
- glass type "E", Young's modulus $E_g = 73\,000$ MPa, Poisson's coefficient $\nu_g = 0.23$, density $\rho_g = 2.6$ g/cm^3.

The sandwich-fabric panel is made up of resin and glass fibers with the volume fraction of glass $f_g = 0.342$. After impregnation of the glass fabric with resin, and after curing, a panel is obtained with skin thickness $t \approx 0.35$ mm and with core thickness c varying in the range $5 \leqslant c \leqslant 20$ mm. Depending on the desired core thickness, various pillar shapes are obtained, and each pillar, made up of many impregnated glass yarns, has a quite variable cross-section geometry. Further work is currently under way to understand the importance, if any, of the actual pillar geometry on the whole sandwich stiffness; in this paper, we have always considered the geometry of the "average" pillar, among those observed by us, and have kept it fixed to that of a cylinder, of circular cross-section of radius $R = 0.2142$ mm. The density of pillars in the core is also variable from one type to the other of sandwich-fabric panel; the most common cases have a density of 25 pillars/cm^2.

During the curing process of this panel, as said above, further layers of impregnated glass fibers are used to thicken the skins, which may reach thicknesses of the order of 1–3 mm. The layers are made by the same glass fibers and resin as the starting panel, with the same volume fractions.

The syntactic foam used to fill the voids in the core of the sandwich-fabric panel is made with the same resin as the panel, and with hollow glass microspheres of the type K1, taken from the industrial batch produced by 3M (3M Italia, 1993). These spheres have average diameter $\Phi = 70$ μm, and ratio a/b between inner and outer radii equal to $a/b = 0.9836$.

We could compare estimates of the stiffness of such sandwich, as given by the assemblage of the techniques illustrated above, with some experimental results obtained in three- and four-point bending by a research group working in parallel with ours (Maier, 1998). The sandwich beam had length $l = 60$ mm, width $B = 30$ mm, skin thickness $t \approx 2$ mm and core thickness $c \approx 11$ mm.

In computing the analytical predictions of the sandwich deflection, we have taken into account the "unwanted" void content in the syntactic foam. In fact, owing to the production modalities of both the syntactic foam and the final sandwich, the sandwich core contained a significant quantity of "unwanted" voids, some of which were clearly visible. The source of the experimental results (Maier, 1998) does not report explicitly the void content relevant to these tests, but it does give some information, apparently about the same sandwich, relevant to different types of testing. According to these data, the average void content of the sandwich core is 30%. Using this information, we could determine the volume fraction of the K1 microspheres present in the syntactic foam filling the core, equal to $f = 0.3877$, and then the effective elastic moduli of the syntactic foam filling the core (see Bardella and Genna (2000), for the formulae allowing to consider the influence of the "unwanted" voids): $E_s = 1190$ MPa and $G_s = 456$ MPa.

Tables 1 and 2 summarize the experimental results given in Maier (1998). For the three-point bending case, the estimate given by the set of analytical tools described in the previous sections furnishes the value $\delta/P = 1.239 \times 10^{-4}$ mm/N, whereas, for the four-point bending case, the analytical estimate is $\delta/P = 0.9213 \times 10^{-4}$ mm/N. The experimental results exhibit some dispersion, whose causes are not easily

Table 1
Three-point bending experimental results (Maier, 1998)

Specimen	Load P (N)	Displacement δ (mm)	δ/P (mm/N) $\times 10^4$
BBPL2	4991	0.93	1.863
BBPL3	4998	0.54	1.080
BBPC2	4998	0.79	1.581
BBPC3	4989	0.94	1.884
AAPL2	4998	1.36	2.721

Table 2
Four-point bending experimental results (Maier, 1998)

Specimen	Load P (N)	Displacement δ (mm)	δ/P (mm/N) $\times 10^4$
BBPL2	7988	0.85	1.064
BBPL3	7991	0.75	0.939
BBPC2	7999	1.27	1.588
BBPC3	8003	0.77	0.962
AAPL2	8000	0.89	1.113

explained (and no attempt at doing that is found in Maier (1998)). The analytical results tend to overestimate the stiffness of the sandwich with reference to all the tests, by an average value of about 32% for the three-point bending case, and of 17% for the four-point bending case. Among the possible causes of this discrepancy, beside the uncertainty about the material and geometry data, is the brittleness of the very thin and large K1 spheres, which might break during the production stage of the syntactic foam, thus increasing the ‘‘weak phase’’ content. Another possible explanation lies in the orthotropy of the skins in their plane, neglected by our model, which assumes, as the Young modulus of the skins, a value coincident with the maximum experimental value (in the weft direction – see Section 4). In any case, considering the complication of the morphology of this sandwich, the obtained results can be considered acceptable from an engineering viewpoint.

6. Design of minimum weight panels

We can now turn to the design of an ‘‘optimum’’ sandwich with respect to its elastic stiffness, considering the possibility of changing all the involved geometry and material parameters. We have explored a choice of both resin and microsphere types, by considering, among the design variables, the resin material parameters and the microsphere geometry, defined, for our purposes, only by the ratio a/b. These variables are all discrete variables, taken from a ‘‘catalog’’. For the resin, we have considered four different materials, whose properties are summarized in Table 3. For the microspheres, we have made reference to the data given by 3M, reported in 3M Italia (1993), and summarized, for the part here of interest, in Table 4. We have always considered a ‘‘perfect’’ syntactic foam, i.e., one in which neither extra voids are present nor the filler particles break during the production process or under the action of the loads.

On the basis of these data, we have explored the influence, on the weight of the sandwich, of a number of basic variables:

- resin material parameters, considered as a discrete variable taken from the above ‘‘catalog’’ (Table 3); we assume to have used the same resin for producing both the sandwich-fabric panel and the syntactic foam;
- microsphere material parameters, considered as a discrete variable taken from the above ‘‘catalog’’ (Table 4);
- volume fraction f of microspheres in the syntactic foam; this is a continuous variable, varied in the interval $0.4 \leqslant f \leqslant 0.6$; the lower limit is based on engineering considerations, whereas the upper one is the technological packing limit of the microsphere–resin mix;

Table 3
Resin properties

Resin type	E_r (MPa)	ν_r (–)	ρ_r (g/cm^3)
1	3700	0.4	1.15
2	2800	0.4	1.18
3	4890	0.4	1.24
4	3500	0.4	1.10

Table 4
Sphere properties, from 3M Italia (1993)

Sphere type	Median diameter (μm)	a/b (−)	ρ_{sph} (g/cm^3)
K1	70	0.9836	0.125
K15	70	0.9802	0.15
K20	60	0.9734	0.20
S22	40	0.9709	0.22
K25	55	0.9667	0.25
S32	45	0.9569	0.32
K37	50	0.9501	0.37
S38	45	0.9489	0.38
K46	50	0.9376	0.46
S60	30	0.9175	0.60

- we have kept fixed the volume fraction of glass fibers in the skins and in the pillars of the core to the value $f_g = 0.342$; this seems to be a technological constraint, which, however, could easily be removed in this theoretical analysis;
- core thickness c, continuously varying in the interval $5 \leqslant c \leqslant 100$ mm;
- skin thickness t, continuous variable, constrained to be smaller than c;
- distance between the pillars in warp direction, b_a, and in weft direction, b_e, continuous variables constrained to be larger than the diameter of one pillar for obvious reasons. As already said, the diameter of one pillar has been kept fixed in this analysis, to the value $2R = 0.4284$ mm; also, this information could easily be considered as a design variable.

We then want to find the set of the preceding values which minimizes the total weight W of a sandwich beam of length l and width B (as schematically shown in Fig. 2)

$$W = gBl\left[2\rho_{sk}t + \frac{\rho_p \pi R^2 c}{b_a b_e} + \rho_s\left(c - \frac{\pi R^2 c}{b_a b_e}\right)\right], \tag{6.1}$$

where g is gravity acceleration, ρ_p is the pillar density (in our case equal to the skin density, and computed on the basis of the resin density indicated in Table 3, the glass density $\rho_g = 2.6$ g/cm^3 and the prescribed volume fraction of glass in the composite, $f_g = 0.342$) and ρ_s is the density of the syntactic foam, computed on the basis of the resin and microsphere densities and on the sphere volume fraction f.

We have here chosen to perform the minimization under the constraint given by Eq. (2.1), i.e., in such a way that the final sandwich has a constant stiffness with respect to the three- and four-point bending test. Extending this constraint to more general loading and geometry conditions requires the knowledge of relationships analogous to Eqs. (2.1)–(2.3), valid, for instance, for bending of plates, etc. The overall beam geometry here considered is $l = 200$ mm and $B = 50$ mm; we have started with the three-point bending case, i.e., by setting $l_p = 0$ in Eqs. (2.2) and (2.3).

The minimization of the total weight (6.1) is obviously non-linear, and cannot be performed by means of standard procedures. We have therefore both studied the objective function as a function of some of the design variables, in order to understand what type of minima are to be sought, and implemented a very simple search algorithm, able to compute a global minimum at a rather high cost. The algorithm requires, at each step, having chosen a set of independent variables, the following computations:

1. elastic moduli of the skins, based on Eqs. (4.10) and (4.15);
2. elastic moduli of the syntactic foam, based on Eqs. (3.1) and (3.2);
3. elastic moduli of the core, based on Eqs. (4.10) and (4.14);
4. bending stiffness of the sandwich (Eq. (2.4));
5. shear stiffness of the sandwich (Eqs. (2.5)–(2.8));

6. solution of the non-linear Eq. (2.1), where the ratio δ/P is assumed as a given datum; in our computations, it has been kept fixed at the value $\delta/P = 0.001$ mm/N. This operation, rather involved owing to the strong nonlinearity of the problem, allows us to eliminate one design variable from the variable set; we have here chosen to eliminate the sandwich thickness t;
7. computation of the weight of the sandwich and selection of the minimum among all those computed previously.

It is obvious that even a single optimization step requires a considerable effort. For this reason, before performing the optimization, we have studied the behavior of the objective function keeping fixed several design variables and varying the remaining ones. The results of this preliminary analysis are shown in Figs. 9–14.

Fig. 9 illustrates the variation of the sandwich weight consequent to the variation of the resin choice, for all the microspheres used in the analysis. In this case, resin material parameters have been considered as continuous variables; the density has been taken as an independent variable and Young's modulus has been correlated linearly to the density, as a first approximation on the safe side, on the basis of results given in Ashby (1989), by means of the following relationship:

$$E_r = 4750\rho_r - 1848, \quad 0.5 \leqslant \rho_r \leqslant 2.5 \tag{6.2}$$

in which Young's modulus is expressed in MPa and the density in g/cm^3. All the other variables (with the exception of t) are set fixed at their "optimum" values, which we will comment about later in this section.

It is apparent from Fig. 9 that the best sandwich, with respect to the stiffness constraint (2.1), is always found using the lightest resin and the lightest microspheres. We will see in a moment that this tendency carries out with respect to more or less all the other design variables.

Fig. 10 illustrates the effect of varying the volume fraction of microspheres in the syntactic foam, for the four resin types considered here and for the lightest and heaviest sphere types. Again, the best sandwich is

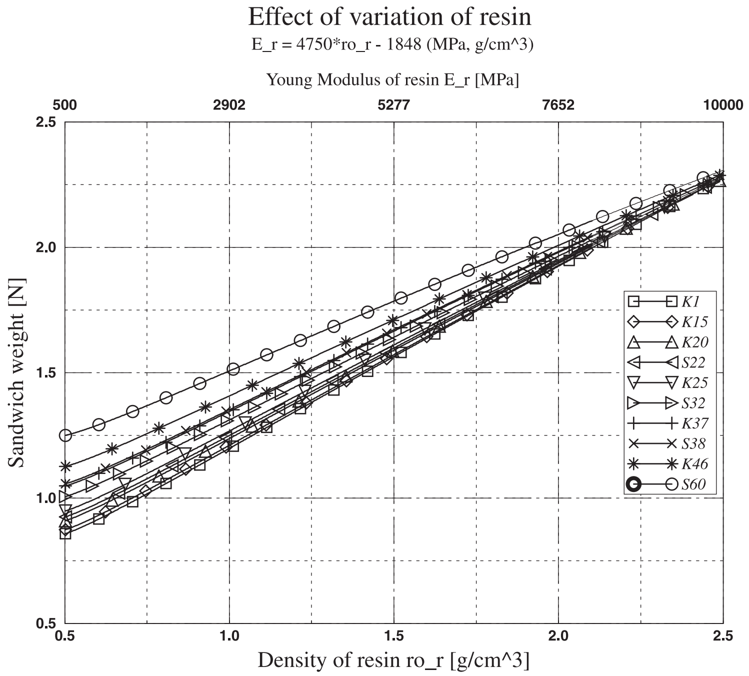

Fig. 9. Effect of variation of resin on the total weight of a sandwich beam of constant stiffness ($E_r = 4750\rho_r - 1848$, MPa g/cm^3).

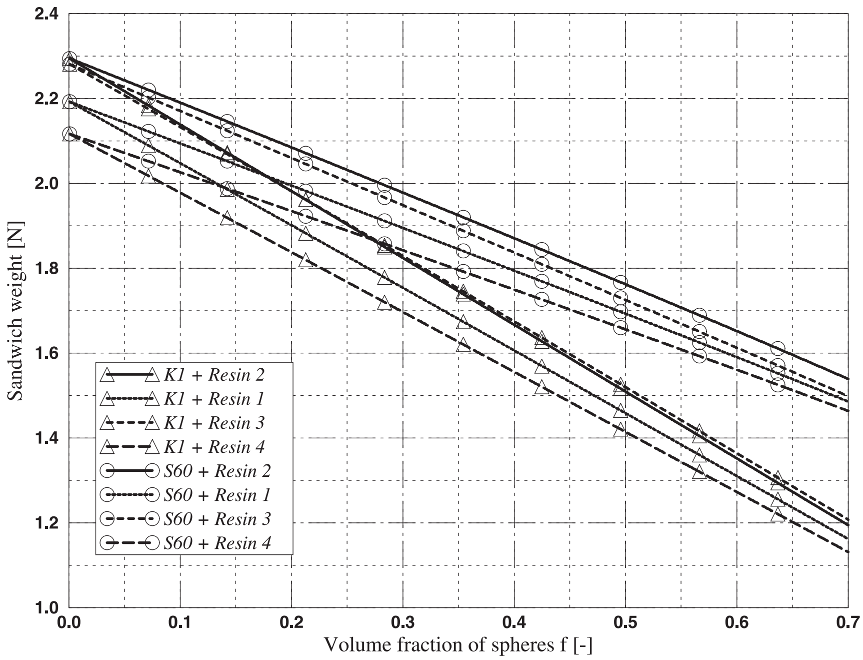

Fig. 10. Effect of variation of the volume fraction of inclusions in the syntactic foam on the total weight of a sandwich beam of constant stiffness.

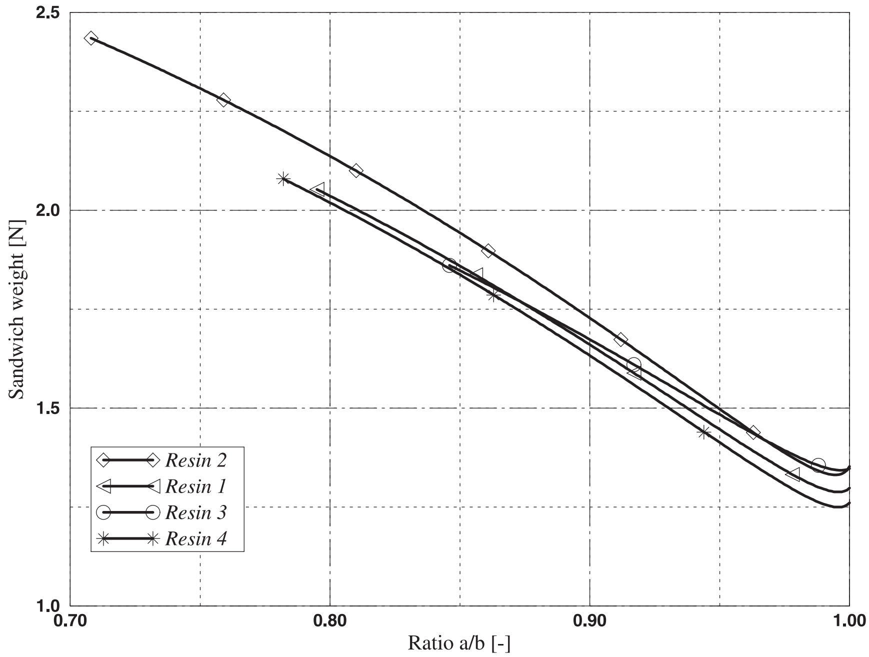

Fig. 11. Effect of variation of the stiffness of inclusions of the syntactic foam on the total weight of a sandwich beam of constant stiffness.

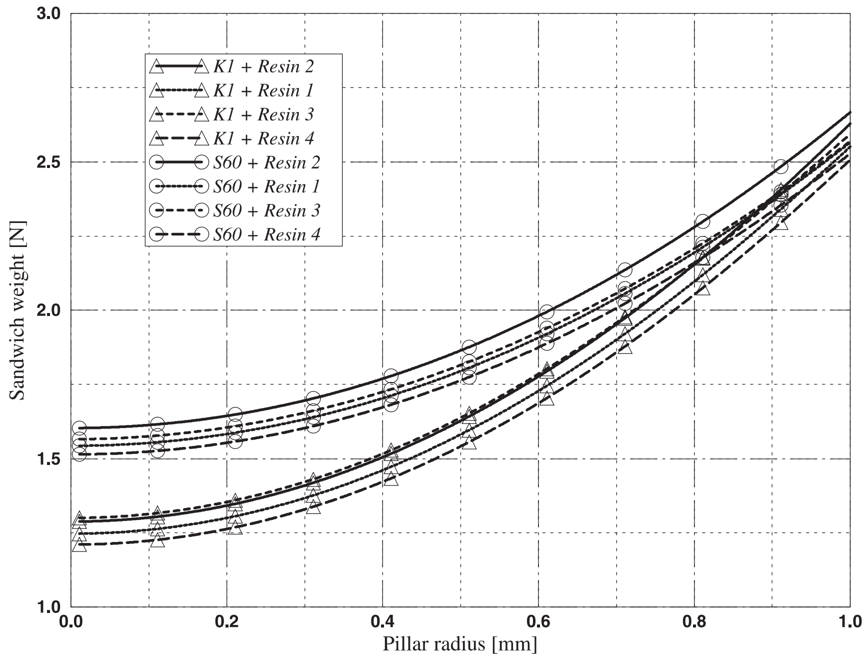

Fig. 12. Effect of variation of the pillar diameter on the total weight of a sandwich beam of constant stiffness.

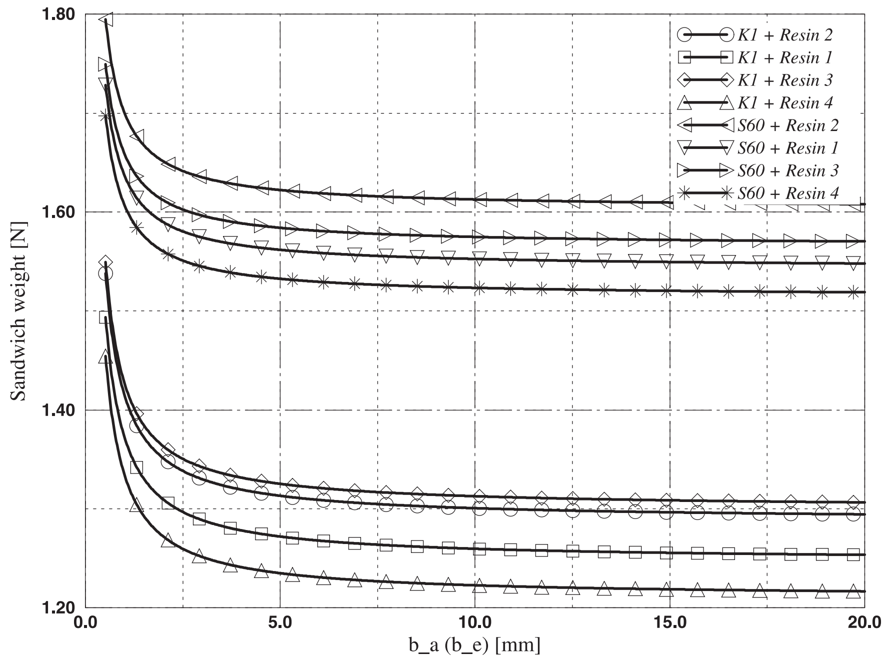

Fig. 13. Effect of variation of the pillar density on the total weight of a sandwich beam of constant stiffness.

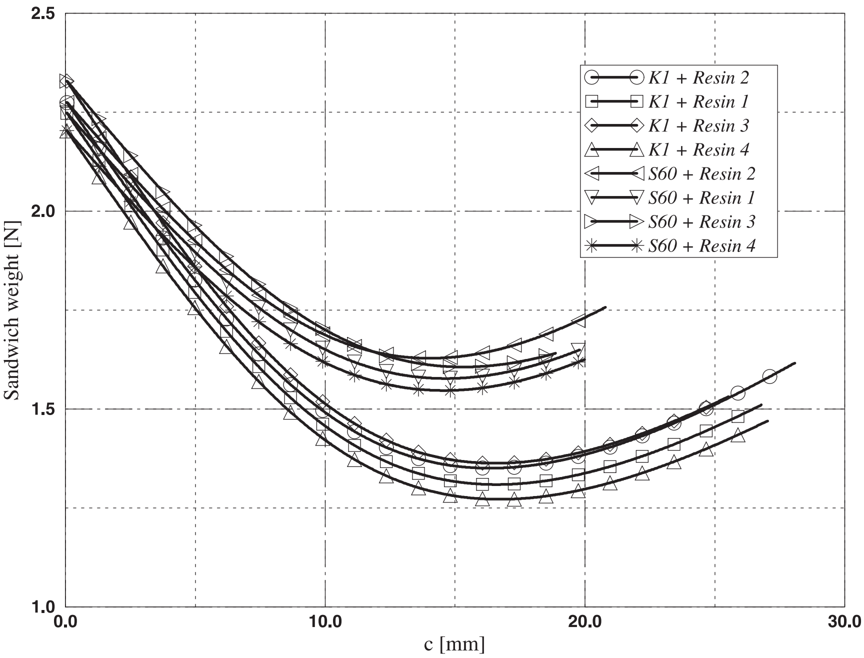

Fig. 14. Effect of variation of the core thickness on the total weight of a sandwich beam of constant stiffness.

always that made up of the lightest ingredients, which means the maximum possible volume fraction of the lightest microspheres in the lightest resin.

Fig. 11 relates to the choice of the microsphere stiffness, as if it were a continuous variable. We have computed the sandwich weight, always under constraint (2.1), as function of the ratio a/b; values of a/b smaller than 0.7 result in unfeasible solutions. It can be seen that, in this case, there is an optimum inclusion stiffness, given by ratios a/b very close to unity, without being equal to 1 (which would mean standard foams, with void inclusions). This indicates that even with the objective of minimum weight, syntactic foams are preferable to standard foams. In our case, of sandwiches which may be employed in underwater applications, standard foams are to be avoided in any case.

In Fig. 12, we have explored the influence of a variable, which has actually been kept fixed in the analysis, for the above-said reasons, i.e., the radius of a pillar in the core. It can be seen that, again, the lightest internal microstructure always gives the best sandwich, in our situation. The chosen value $R = 0.2142$ mm is not optimal, but definitely close to it.

Fig. 13 shows the influence on the sandwich weight of the pillars density in the core. It is readily seen that the smaller the number of pillars, the lighter the resulting structure, even if the stiffness constraint (2.1) is always prescribed. This conclusion, however, cannot be taken into too much consideration, owing to the delamination problems mentioned in Section 1, which will be briefly re-addressed later on. In our analysis, we have set, as a lower limit to the pillar density, the value of 25 pillars/cm^2, which means, for equal spacing in the warp and weft directions, $b_a = b_e = 2$ mm.

Finally, Fig. 14 shows the effect of the variation of the core thickness (related to that of the skin thickness by constraint (2.1)) on the sandwich weight. Despite all the non-linearities of the problem, we have here a well defined single, absolute minimum for all resins and microspheres examined.

On the basis of these results, it is possible to set up a very much reduced optimization procedure of the sandwich than that previously illustrated. One can in fact choose the lightest resins and microspheres, use the highest possible volume fraction of spheres and use the lightest possible sandwich-fabric panel (the smallest pillar density compatible with delamination risks), and, thus, eliminate several design variables from his problem, which is reduced to two variables only, the thicknesses of the core c and of the skins t.

It is then possible to operate at different levels of approximation. If one computes the stiffness of the sandwich as indicated (i.e., using Eqs. (2.1) and (2.5)–(2.8)), one obtains the "exact" result. A first approximation, which we indicate as approximation 1, may consist of choosing the much simpler expression (2.10) for the shear stiffness. In this case, it is possible to express the variable t as function of c, using the approximation $d \approx c$ and always considering the three-point bending case $l_p = 0$, through Eq. (2.1), obtaining

$$t(c) = -\frac{c}{2} - \frac{\sqrt[3]{cE_{sk}^2\left(l - 4BcG_c\frac{\delta}{P}\right)^2\left[c^2lE_{sk} - c^2lE_c - l^3G_c + 4Bc^3E_cG_c\frac{\delta}{P} - 4Bc^3E_{sk}G_c\frac{\delta}{P}\right]}}{2E_{sk}\left(l - 4BcG_c\frac{\delta}{P}\right)}. \tag{6.3}$$

Inserting expression (6.3) into Eq. (6.1) and equating to zero the derivative of the result with respect to c, one obtains a non-linear equation in c, which can be solved via Newton's method. We will comment on the results in a moment.

A cruder approximation (approximation 2) can be obtained by adopting both the thin skins and the antiplane core expressions for both the bending and the shear stiffness, i.e., Eqs. (2.9) and (2.10). Again, setting $d \approx c$, $l_p = 0$ and solving Eq. (2.1) for t, one gets

$$t(c) = \frac{G_c l^3}{6E_{sk}c\left(4BcG_c\frac{\delta}{P} - l\right)} \tag{6.4}$$

and, inserting this result into Eq. (6.1) and making it stationary with respect to c, one obtains a fourth-order algebraic equation in c, which can be solved in the closed form.

We can finally discuss the obtained results. As said, the best sandwich is essentially the lightest one, even if not necessarily made with the stiffest materials. The set of "optimum" parameters, as given by the "exact" optimization procedure, is the following:

- resin type 4 of Table 3;
- microspheres type K1 of Table 4;
- $c^{opt} = 16.21$ mm;
- $t^{opt} = 1.16$ mm;
- $f^{opt} = 0.6$ (maximum value allowed during the analysis);
- $b_a^{opt} = b_e^{opt} = 2.0$ mm (maximum values allowed during the analysis);
- skin properties: $E_{sk} = 16060$ MPa; $G_{sk} = 4852$ MPa;
- syntactic foam properties: $E_s = 2076$ MPa; $G_s = 783$ MPa;
- core properties: $E_c = 2160$ MPa; $G_c = 811$ MPa;
- beam stiffnesses: $D = 0.179 \times 10^9$ N mm^2; $(GA^*) = 0.723 \times 10^6$ N.

The sandwich weight is then $W_{min} = 1.249$ N, which corresponds to a sandwich density $\rho = 0.687$ g/cm^3. To give an idea of the savings in weight, with respect to a conventional beam, we may say that a beam of same length and width, made by a standard fiberglass reinforced resin ($E = 16000$ MPa, $\rho = 1.8$ g/cm^3), in order to exhibit the same stiffness needs a height $h = 14$ mm and a weight of about 2.5 N.

The two approximate optimization methods described above, applied by using the homogenized Young's moduli of the "optimum" sandwich, yield the following results:

1. *approximation 1:* $t^{opt} = 1.13$ mm, $c^{opt} = 16.4$ mm;
2. *approximation 2:* $t^{opt} = 1.452$ mm, $c^{opt} = 17.535$ mm.

Only the first result can be considered reasonable from the engineering viewpoint; it is clear that the most important error is introduced, for this sandwich, by the assumption of thin skins and antiplane core in the bending stiffness D (approximation 2), and not by the use of the simple formula (2.10) for the shear stiffness instead of the more complex Eqs. (2.5)–(2.8).

We conclude this section by adding the results for the four-point bending case, in order to give an example of how the loading condition affects the design of the sandwich. By setting $l_p = l/3$ as the distance between the points of application of the concentrated forces in Eqs. (2.2) and (2.3), one obtains the following "exact" results:

- resin type 4 of Table 3;
- microspheres type K1 of Table 4;
- $c^{opt} = 15.205$ mm;
- $t^{opt} = 1.1$ mm;
- $f^{opt} = 0.6$ (maximum value allowed during the analysis);
- $b_a^{opt} = b_e^{opt} = 2.0$ mm (maximum values allowed during the analysis);
- beam stiffnesses: $D = 0.149 \times 10^9$ N mm^2; $(GA^*) = 0.679 \times 10^6$ N.

The approximate results exhibit the same type of error as in the three-point bending case.

The sandwich weight in this case is $W_{min} = 1.175$ N, which corresponds to a sandwich density $\rho = 0.688$ g/cm^3. The same "conventional" fiberglass reinforced resin beam described above, designed to exhibit the same stiffness in four-point bending, has height $h = 12.87$ mm and weighs 2.32 N. In both loading cases, the saving in weight is of the order of 50%.

7. Open issues and conclusions

The optimization procedure outlined in this work is only a first step in the design of a sandwich, which may be produced for a large-scale utilization. There is a number of problems still to be addressed, and even in the elastic range, the conclusions indicated by this first analysis cannot be accepted without further scrutiny.

We can immediately observe that optimization is possible with respect to several other loading and geometry conditions; it is well known that different optimal microstructures are required depending on the stress type – axial loading vs. beam bending or plate bending or fully three-dimensional stress states. The extension of the analysis procedure followed here to all these loading conditions does not pose, however, conceptual difficulties.

Another issue to be addressed is the influence of the pillar shape on the overall sandwich behavior. This parameter is of the utmost importance in the behavior of the sandwich-fabric panel alone, as shown in Van Vuure (1997). It is however very likely that in the filled sandwich, this aspect loses some significance; in fact, in the sandwich, owing also to the extremely low volume fraction of the pillars (of the order of 5%), the core properties, with reference to the sandwich bending and shear stiffness, are essentially those of the syntactic foam (this is confirmed also by the very small difference between the elastic moduli of the foam and those of the homogenized core given as "optimal" in Section 6). Such an analysis, in any case, would require microstructural techniques more sophisticated than the simple homogenization methods used in this work.

As said, the conclusions reached with respect to the elastic stiffness cannot be accepted without some reservation. The optimum sandwich, with reference to its elastic stiffness only, is made up of the lightest ingredients, i.e., the lightest resin and the maximum possible volume fraction of the lightest K1 microspheres, with the lowest possible density of pillars in the core. All these choices have of course some drawbacks, which need careful examination when designing a sandwich. The light K1 glassy microspheres, for instance, tend to break during the production process of the syntactic foam, thus introducing both a degradation of the stiffness and a path for permeability. This reason alone is sufficient to reconsider such choice.

The quest for a light core may suggest the use of non-glassy inclusions, or even to mix particulate inclusions with air. These are possible solutions, which need careful examination, outside the scope of this work. An alternative solution may include the use of phenolic microspheres, as done in Bunn and Mottram (1993); we have presently no experience of such materials.

Also, the density of pillars creates conflicting indications. As said earlier, the pillars give almost no contribution, in bending and in shear, to the elastic stiffness, but they are essential to guarantee the absence of risk of delamination between the skins and the core. This brings us to issues of strength, not addressed here but currently under investigation. The only result obtained so far with reference to delamination indicates that, for standard beam applications, a minimum density of pillars (i.e., the 25 pillars/cm^2 already used in the production of the prototype sandwich) is more than enough to guarantee the ability of carrying the shear stress arising in the three-point bending (Bardella and Genna, 1999).

Nothing has been done in the field of non-linear analysis of both the syntactic foam and of the final sandwich, whose internal complexity poses a rather formidable problem. We are aware of some research done on the non-linear behavior of woven fabric composites (see, for instance, Tabiei and Jiang (1999) and references cited therein), but in all cases on two-dimensional fabric. Also, the issue of strength of syntactic foams alone is important; experiments, done on the described syntactic foams filled with hollow glassy spheres, indicate a brittle behavior both in tension and in compression, coupled with relatively high strength (of the order of 100 MPa in compression and 50 MPa in tension). Different failure mechanisms have been observed, related to different choices of spheres among the standard industrial batch of Table 4. All these topics are the subject of work currently under way.

The results described in Section 6 indicate that despite the remarkable internal complexity of this sandwich, its minimum weight design, with respect to the elastic behavior only and with reference to specific geometry and loading conditions, does not require sophisticated optimization techniques. This conclusion should help practicing engineers in designing such sandwiches for large-scale production.

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Appendix A

The coefficients F_1, F_2, F_3, F_4 and H_4 of Eq. (3.1) are defined as follows, where G_i and ν_i are the shear modulus and the Poisson ratio of the inclusion wall and G_m and ν_m are the analogous elastic constants of the matrix. If a, b and s are the radii of the composite sphere, as illustrated in Fig. 5, define

$$C_1 = \frac{21}{5}\frac{b}{a} - 6\nu_i\frac{b^3}{a^3} + \frac{3}{20}(7 + 5\nu_i)\frac{a^4}{b^4}, \tag{A.1}$$

$$C_2 = \frac{2}{5}(7-5\nu_i)\frac{b}{a} + (5-4\nu_i)\frac{a^2}{b^2} - \frac{9}{5}\frac{a^4}{b^4}, \tag{A.2}$$

$$C_3 = \frac{21}{5}\frac{b}{a} - (7-4\nu_i)\frac{b^3}{a^3} - \frac{1}{10}(7+5\nu_i)\frac{a^4}{b^4}, \tag{A.3}$$

$$C_4 = \frac{2}{5}(7-5\nu_i)\frac{b}{a} + 2(1-2\nu_i)\frac{a^2}{b^2} + \frac{6}{5}\frac{a^4}{b^4}, \tag{A.4}$$

$$C_5 = \frac{42}{5} + 6\nu_i\frac{b^2}{a^2} - \frac{6}{5}(7+5\nu_i)\frac{a^5}{b^5}, \tag{A.5}$$

$$C_6 = \frac{4}{5}(7-5\nu_i) - 4(5-\nu_i)\frac{a^3}{b^3} + \frac{72}{5}\frac{a^5}{b^5}, \tag{A.6}$$

$$C_7 = \frac{21}{5} - (7+2\nu_i)\frac{b^2}{a^2} + \frac{2}{5}(7+5\nu_i)\frac{a^5}{b^5}, \tag{A.7}$$

$$C_8 = \frac{2}{5}(7-5\nu_i) + 2(1+\nu_i)\frac{a^3}{b^3} - \frac{24}{5}\frac{a^5}{b^5}, \tag{A.8}$$

then

$$D_1 = (C_5(C_4-C_2) + C_6(C_1-C_3))\frac{b}{a} - 2\frac{G_m}{G_i}(C_1C_4 - C_2C_3), \tag{A.9}$$

$$D_2 = ((C_2C_5 - C_1C_6)(7-4\nu_m) + 6(C_3C_6 - C_4C_5)\nu_m)\frac{b^3}{a^3} - 6\frac{G_m}{G_i}\nu_m(C_1C_4 - C_2C_3)\frac{b^2}{a^2}, \tag{A.10}$$

$$D_3 = (C_5(2C_2+3C_4) - C_6(2C_1+3C_3))\frac{a^4}{b^4} + 24\frac{G_m}{G_i}(C_1C_4 - C_2C_3)\frac{a^5}{b^5}, \tag{A.11}$$

$$\begin{aligned} D_4 = {} & ((C_4C_5 - C_3C_6)(5-4\nu_m) + 2(C_1C_6 - C_2C_5)(1-2\nu_m))\frac{a^2}{b^2} \\ & + 4\frac{G_m}{G_i}(5-\nu_m)(C_1C_4 - C_2C_3)\frac{a^3}{b^3}, \end{aligned} \tag{A.12}$$

$$D_5 = (C_7(C_4-C_2) + C_8(C_1-C_3))\frac{b}{a} - \frac{G_m}{G_i}(C_1C_4 - C_2C_3), \tag{A.13}$$

$$D_6 = ((C_2C_7 - C_1C_8)(7-4\nu_m) + 6(C_3C_8 - C_4C_7)\nu_m)\frac{b^3}{a^3} + \frac{G_m}{G_i}(7+2\nu_m)(C_1C_4 - C_2C_3)\frac{b^2}{a^2}, \tag{A.14}$$

$$D_7 = (C_7(2C_2+3C_4) - C_8(2C_1+3C_3))\frac{a^4}{b^4} - 8\frac{G_m}{G_i}(C_1C_4 - C_2C_3)\frac{a^5}{b^5}, \tag{A.15}$$

$$\begin{aligned} D_8 = {} & ((C_4C_7 - C_3C_8)(5-4\nu_m) + 2(C_1C_8 - C_2C_7)(1-2\nu_m))\frac{a^2}{b^2} \\ & - 2\frac{G_m}{G_i}(1+\nu_m)(C_1C_4 - C_2C_3)\frac{a^3}{b^3}, \end{aligned} \tag{A.16}$$

then again

$$H_1 = \frac{D_3D_6 - D_2D_7}{D_1D_7 - D_3D_5}\frac{s}{a} - 6v_m\frac{s^3}{a^3} + 3\frac{D_2D_5 - D_1D_6}{D_1D_7 - D_3D_5}\frac{a^4}{s^4}, \tag{A.17}$$

$$H_2 = \frac{D_3D_8 - D_4D_7}{D_1D_7 - D_3D_5}\frac{s}{a} + (5 - 4v_m)\frac{a^2}{s^2} + 3\frac{D_4D_5 - D_1D_8}{D_1D_7 - D_3D_5}\frac{a^4}{s^4}, \tag{A.18}$$

$$H_3 = \frac{D_3D_6 - D_2D_7}{D_1D_7 - D_3D_5}\frac{s}{a} - (7 - 4v_m)\frac{s^3}{a^3} - 2\frac{D_2D_5 - D_1D_6}{D_1D_7 - D_3D_5}\frac{a^4}{s^4}, \tag{A.19}$$

$$H_4 = \frac{D_3D_8 - D_4D_7}{D_1D_7 - D_3D_5}\frac{s}{a} + 2(1 - 2v_m)\frac{a^2}{s^2} - 2\frac{D_4D_5 - D_1D_8}{D_1D_7 - D_3D_5}\frac{a^4}{s^4}, \tag{A.20}$$

$$H_5 = 2\frac{D_3D_6 - D_2D_7}{D_1D_7 - D_3D_5} + 6v_m\frac{s^2}{a^2} - 24\frac{D_2D_5 - D_1D_6}{D_1D_7 - D_3D_5}\frac{a^5}{s^5}, \tag{A.21}$$

$$H_6 = 2\frac{D_3D_8 - D_4D_7}{D_1D_7 - D_3D_5} - 4(5 - v_m)\frac{a^3}{s^3} - 24\frac{D_4D_5 - D_1D_8}{D_1D_7 - D_3D_5}\frac{a^5}{s^5}, \tag{A.22}$$

$$H_7 = \frac{D_3D_6 - D_2D_7}{D_1D_7 - D_3D_5} - (7 + 2v_m)\frac{s^2}{a^2} + 8\frac{D_2D_5 - D_1D_6}{D_1D_7 - D_3D_5}\frac{a^5}{s^5}, \tag{A.23}$$

$$H_8 = \frac{D_3D_8 - D_4D_7}{D_1D_7 - D_3D_5} + 2(1 + v_m)\frac{a^3}{s^3} + 8\frac{D_4D_5 - D_1D_8}{D_1D_7 - D_3D_5}\frac{a^5}{s^5}, \tag{A.24}$$

and finally,

$$F_1 = \left((H_1 - H_3)\frac{H_6H_1 - H_5H_2}{H_1H_4 - H_3H_2} + H_5\right)\frac{s}{a}, \tag{A.25}$$

$$F_2 = \left((2H_1 + 3H_3)\frac{H_6H_1 - H_5H_2}{H_1H_4 - H_3H_2} - 3H_5\right)\frac{a^4}{s^4}, \tag{A.26}$$

$$F_3 = \left((H_1 - H_3)\frac{H_8H_1 - H_7H_2}{H_1H_4 - H_3H_2} + H_7\right)\frac{s}{a}, \tag{A.27}$$

$$F_4 = \left((2H_1 + 3H_3)\frac{H_8H_1 - H_7H_2}{H_1H_4 - H_3H_2} - 3H_7\right)\frac{a^4}{s^4}. \tag{A.28}$$

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